POTW #02

The PH-SL0 Droid

A solid sphere of radius R is truncated horizontally at a distance $z = R/2$ above its geometric center. A solid right circular cone of height H is attached to the sphere such that its base coincides with the flat cut. The combined object has uniform density ρ, total mass M, and rests upright on a horizontal floor.

Part (a): Find the maximum height H_c of the cone in terms of R such that the upright orientation is a stable equilibrium.

Part (b): Let $H = R/2$ and assume the floor has sufficient friction to prevent slipping. The object is displaced by a small angle from its upright equilibrium and released from rest. Find the angular frequency ω of the oscillations. [Hint: the moment of inertia about its center of mass is $I_{\text{cm}} = \frac{3}{8}MR^2$.]

Part (c): If the object from Part (b) is instead placed on a frictionless horizontal plane, find the new angular frequency ω' of small oscillations.